Exercise: solving matrices

- Consider the system of equations:

$$4x + 5y = 3$$
$$x - 3z = -10$$
$$3x - y + 2z = 15$$

- Express this system as a matrix and solve by hand
- Express this same system in MATLAB and solve

Eigenvalue Problems

$$\boldsymbol{A}\boldsymbol{x} = \lambda \boldsymbol{x}$$

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$

$$\lambda \rightarrow eigenvalues, \boldsymbol{x} \rightarrow eigenvectors$$

- Just like with oscillators you will spend the next 3 years of your life solving this problem. **GET GOOD AT IT!**

Solution method

$$\boldsymbol{A}\boldsymbol{x} = \lambda\boldsymbol{x}$$

$$(\boldsymbol{A} - \lambda\boldsymbol{I})\boldsymbol{x} = \boldsymbol{0}$$

$\det(\boldsymbol{A} - \lambda\mathbf{I}) = 0$ (some polynomial usually)

Solve for λ's then plug back into second equation and solve for $\boldsymbol{x}$ to find eigenvectors

Solution method

- Solve for the eigenvalues of the following system by hand

$$\begin{bmatrix} 3 & -1 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

- What do these eigenvectors represent?

Exercise: Second Order ODE

- Consider the equation of motion for this system:

$$\frac{d^2x}{dt^2} = -ax - bx^3 \; - c\,\frac{dx}{dt}$$

- Rewrite this system as an eigenvalue problem and solve for
- a = 5, b = 2, c = 0.5
- $x_0 = 1, \frac{dx_0}{dt} = 0$

Exercise: Second Order ODE

- Consider the equation of motion for this system:

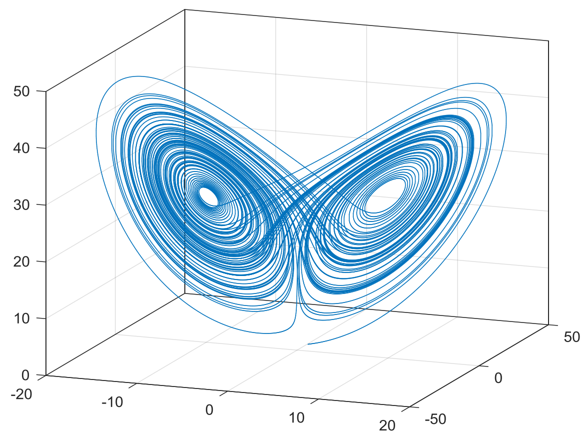
$$\frac{d^2x}{dt^2} = -ax - bx^3 \; - c\,\frac{dx}{dt}$$

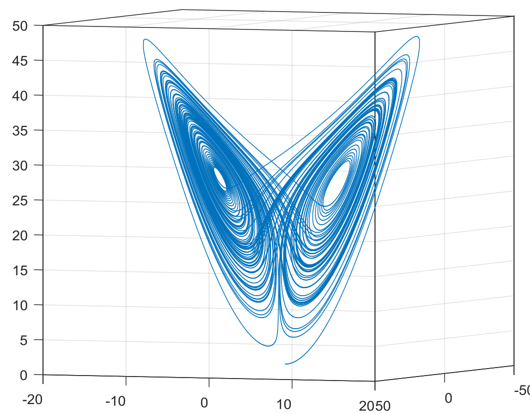
- Find the equilibrium points for this system
- Use the eigs() function to find the eigenvalues of the Jacobian for each equilibrium point
- Are these equilibrium points stable or not?

Exercise: Systems of ODEs

- Let's consider the chaotic system defined by the Lorentz equations

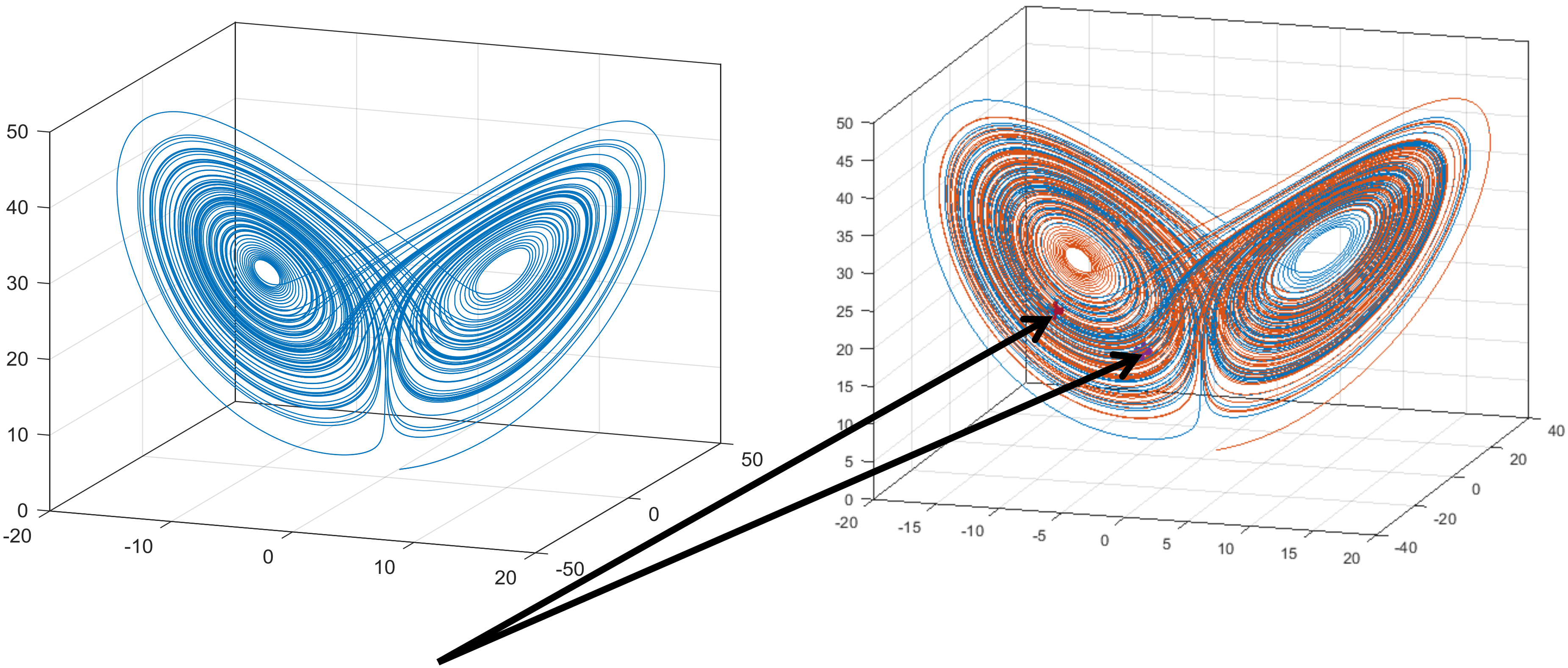
$$\frac{dx}{dt} = \sigma(y - x)$$
$$\frac{dy}{dt} = x(\rho - z) - y$$
$$\frac{dz}{dt} = xy \; - \beta z$$





- Use your Runge-Kutta function to generate solutions to this 3-D system over time interval t = [0 100]. Use step-size 0.001, and initial position x=1,y=1,z=1
- Use values
  - $\sigma = 10$
  - $\rho = 28$
  - $\beta = \frac{8}{3}$
- This is the classic example of a Chaotic system. Try setting x(1) = 1.000001 and re-calculating…

Exercise: Systems of ODEs



- Even a small change to starting conditions produces massive divergence in final position!
- *Chaotic* systems are defined by this property.
- Why can't we model turbulence, weather, etc? …Chaos!

Challenge

- You are modeling the motion of a particle in the atmosphere
    - (This is what the Lorenz equation actually describes)

- You can measure the x, y, z position of the particle to an accuracy of 1 part-per-million
- *i.e.* $x = x * (1 \pm 0.57E - 6)$
  $y = y * (1 \pm 0.57E - 6)$
  $z = z * (1 \pm 0.57E - 6)$

- Knowing that the initial position is x = 1, y = 1, z = 1, **determine how far into the future you can accurately predict the position of the particle**.